TRINITY COLLEGE

Semester One Examination, 2016

Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	53	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	151	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the Instructions to Candidates. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

Consider the function $f(x) = x^2 - 4x$.

(a) Explain why it is necessary to restrict the natural domain of f in order that its inverse is also a function. (1 mark)

(b) State a minimal restriction to the domain of f that includes $x = 3$, and then use this restriction to show that $f^{-1}(x) = 2 + \sqrt{x+4}$. (4 marks)

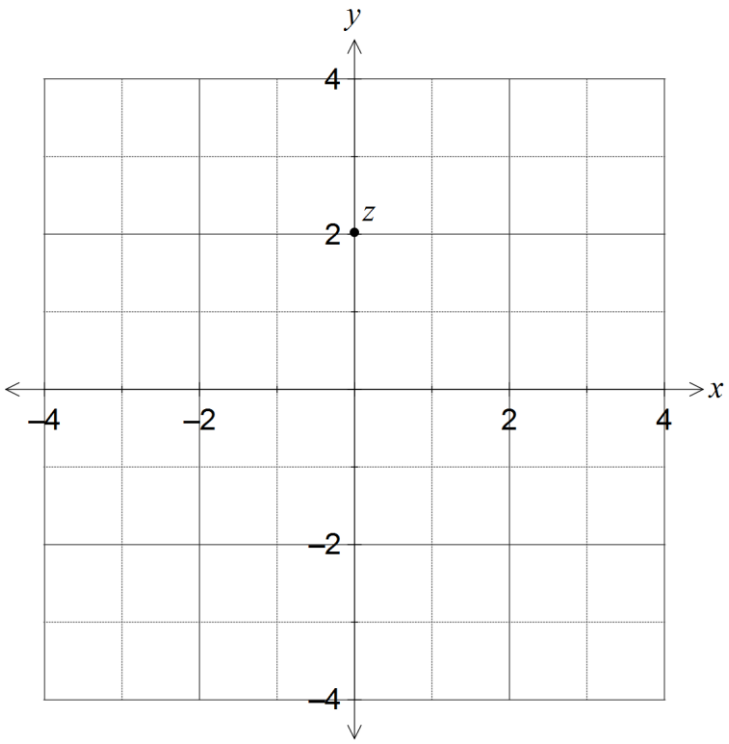
Question 9 **(5 marks)**

(a) Let z be a non-zero complex number located in the complex plane. Describe the linear transformation(s) required to transform z to each of the following locations:

(i) $2z$. (1 mark)

(ii) $i^3 z$. (1 mark)

(b) Consider the complex number z shown in the Argand diagram below. Add to the diagram the location of u, v and w where $u = (1+i)z$, $v = z \cdot \overline{z}$ and $w = \sqrt{z}$. (3 marks)



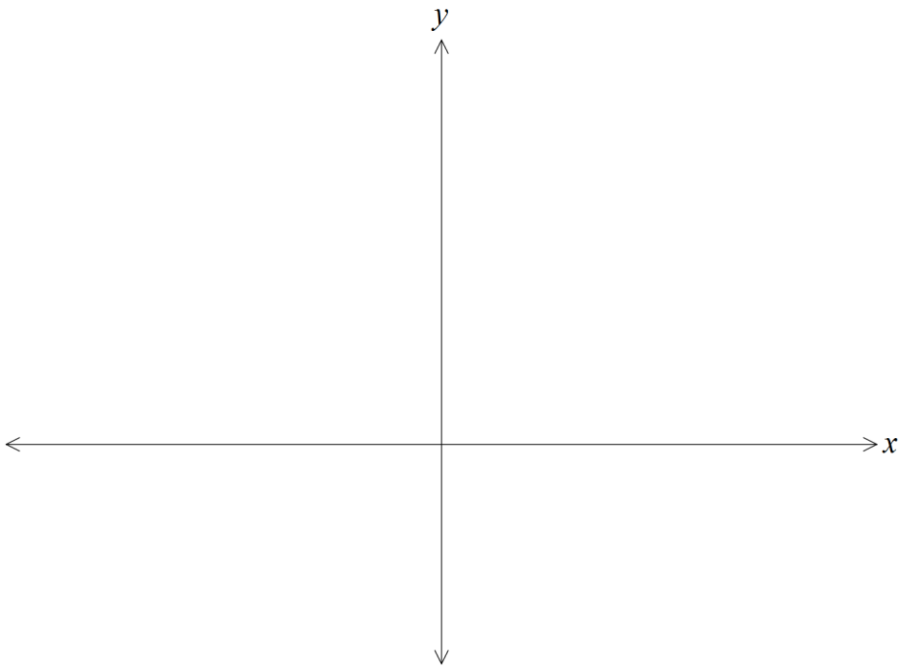
Question 10 **(8 marks)**

Two functions are given by $f(x) = 2\sqrt{x+1}$ and $g(x) = x^2 - 2x$.

(a) Determine $g \circ f(x)$ and state the domain and range of this composite function. (3 marks)

(b) Show that the composite function $f \circ g(x)$ is defined for $x \in \mathbb{R}$. (3 marks)

(c) Sketch the graph of $y = f \circ g(x)$ on the axes below. (2 marks)



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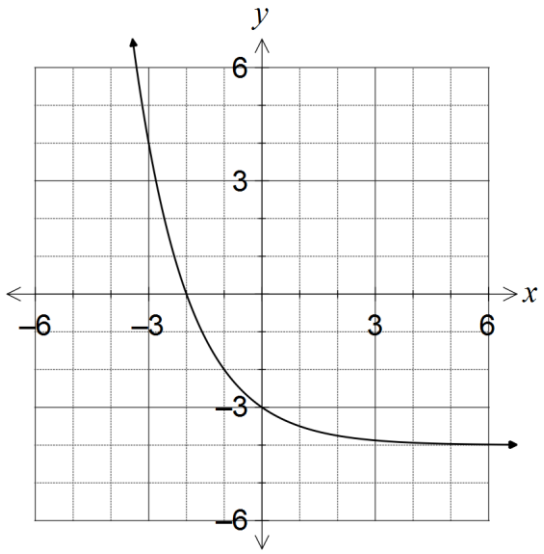
Question 11 **(12 marks)**

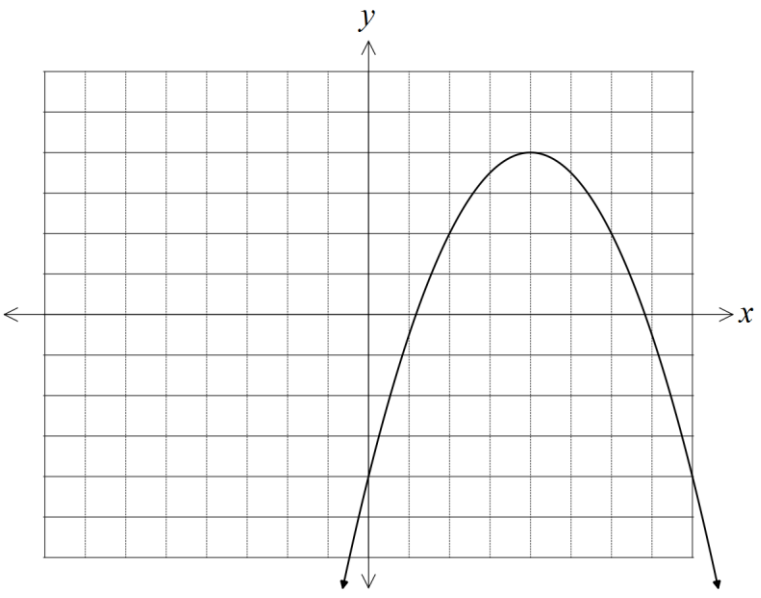
(a) The graph of $y = f(x)$ is shown below.



(i) What feature of the graph suggests that the inverse of f is a function? (1 mark)

(ii) On the same axes, sketch the graph of the inverse of f, $y = f^{-1}(x)$. (3 marks)

(b) The graph of $y = g(x)$ is shown below.
On the same axes, sketch the graph of $y = |g(|x|)|$. (3 marks)



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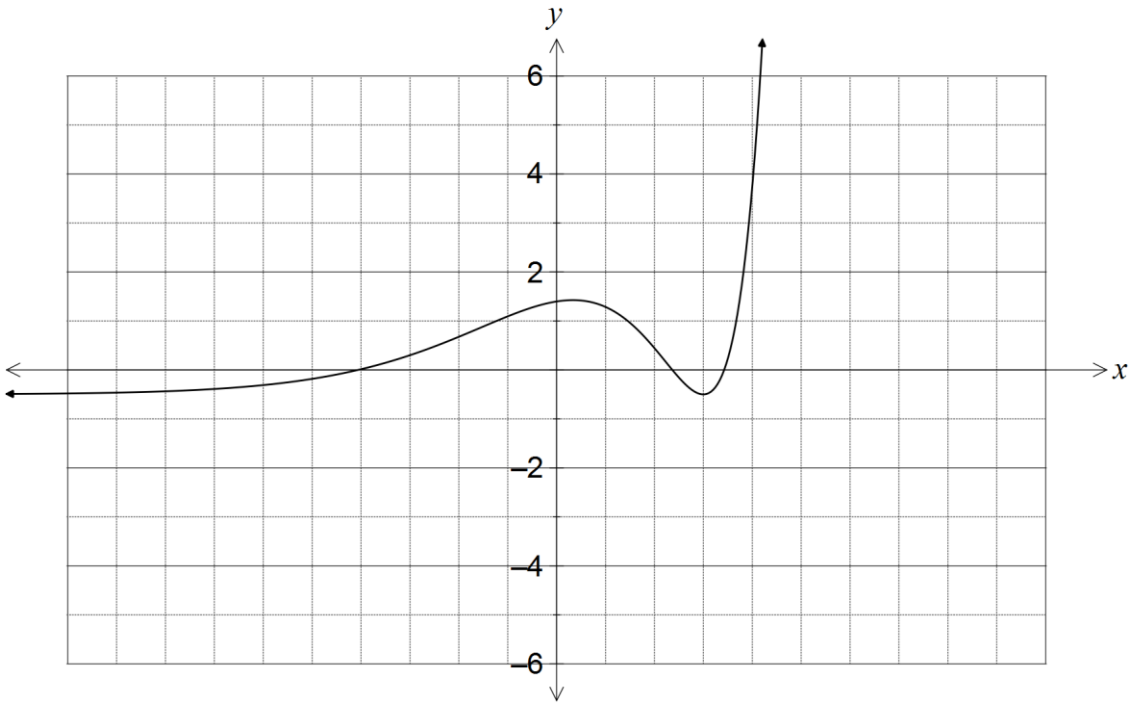
(c) The graph of $y = h(x)$ is shown below. As $x \to -\infty,\ h(x) \to -0.5$. On the same axes, sketch the graph of $y = \dfrac{1}{h(x)}$, clearly indicating all vertical and horizontal asymptotes.

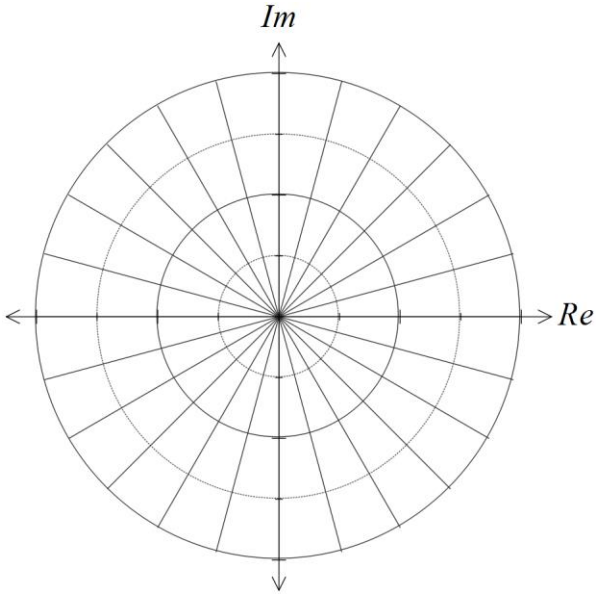
(5 marks)



Question 12 **(8 marks)**

(a) Determine all roots of the equation $z^6 + 8i = 0$, expressing them in exact polar form $rcis\theta$ where $r > 0$ and $-\pi < \theta \leq \pi$. (5 marks)

(b) Show all solutions of the equation in part (a) on the Argand diagram below. (3 marks)



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Question 13 **(7 marks)**

Two small bodies, A and B, simultaneously leave their initial positions of $\mathbf{i}+4\mathbf{j}-25\mathbf{k}$ and $16\mathbf{i}+\mathbf{j}-2\mathbf{k}$, and move with constant velocities of $4\mathbf{i}+\mathbf{j}+5\mathbf{k}$ and $-\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ respectively.

(a) Determine whether the paths of the bodies cross or if the bodies meet. (4 marks)

(b) At the same time, a third small body, C, leaves its initial position, passes through the origin and crosses the path of body A. If C moves with a steady velocity of $5a\mathbf{i}+5\mathbf{j}+a\mathbf{k}$, determine the value of the constant a. (3 marks)

Question 14 **(9 marks)**

The function f is defined by $f(x) = \dfrac{x^2 - 1}{x^2 - 3x + 2}$.

(a) Determine the natural domain and range of $f(x)$. (4 marks)

(b) Show that the function has no stationary points. (2 marks)

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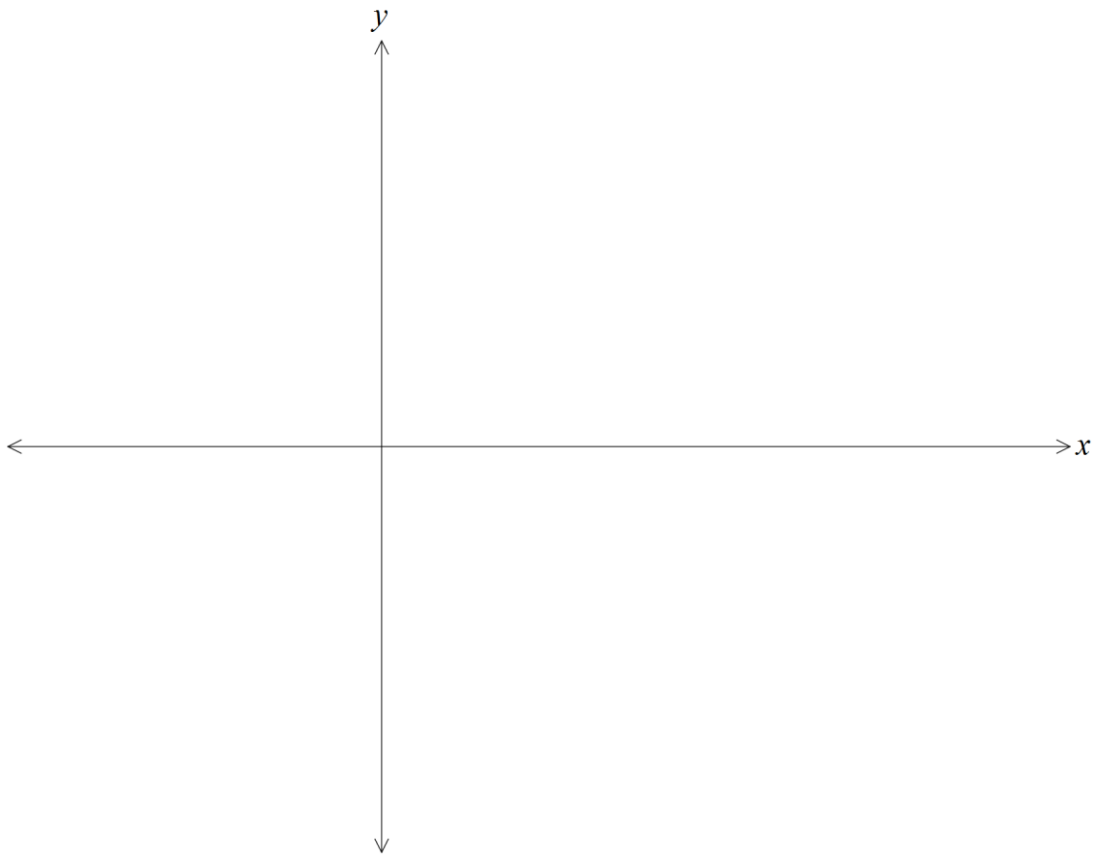
(c) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



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Question 15 **(8 marks)**

Given the two complex numbers $w = r(\cos\theta + i\sin\theta)$ and $z = s(\cos\phi + i\sin\phi)$, determine the following in terms of the non-zero constants r, s, θ and ϕ:

(a) $\arg(\overline{z})$. (1 mark)

(b) $\left|\frac{i}{w^2}\right|$. (2 marks)

(c) $|(1-i)wz|$. (2 marks)

(d) $\arg\left(-\frac{z}{iw}\right)$. (3 marks)

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Question 16 **(7 marks)**

Consider the three vectors $\mathbf{a} = \langle 2,1,-3 \rangle$, $\mathbf{b} = \langle -3,5,-2 \rangle$ and $\mathbf{c} = \langle 2,-4,1 \rangle$.

(a) Prove that the three vectors do not lie in the same plane. (4 marks)

(b) Determine the value(s) of the constant a if the vector $\langle a^2, a, a-3 \rangle$ lies in the same plane as vectors $\mathbf{a}$ and $\mathbf{b}$. (3 marks)

Question 17 **(9 marks)**

Let the complex number $z = \cos\theta + i\sin\theta$.

(a) Show that $\frac{1}{z} = \cos\theta - i\sin\theta$. (2 marks)

(b) Show that $z^3 - \frac{1}{z^3} = 2i\sin 3\theta$. (2 marks)

(c) Determine $\text{Im}\left(z^3 - \frac{1}{z^3}\right)$ in terms of $\sin\theta$ and $\cos\theta$.

(3 marks)

See next page

(d) Express $\sin^3\theta$ in terms of $\sin\theta$ and $\sin 3\theta$. (2 marks)

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Question 18 **(13 marks)**

The velocity vector of a particle at time *t* seconds is $\mathbf{v}(t) = 3\mathbf{i} - \frac{3}{t^2}\mathbf{j}$, for $t \geq 1$. When $t = 1$, the particle has position vector $2\mathbf{j}$.

(a) Calculate the exact speed of the particle when $t = 2$. (2 marks)

(b) Determine the acceleration vector of the particle and comment on its direction. (2 marks)

(c) Determine the position vector of the particle for $t \geq 1$. (2 marks)

(d) Derive the Cartesian equation of the path of the particle in the form $y = f(x)$. (2 marks)

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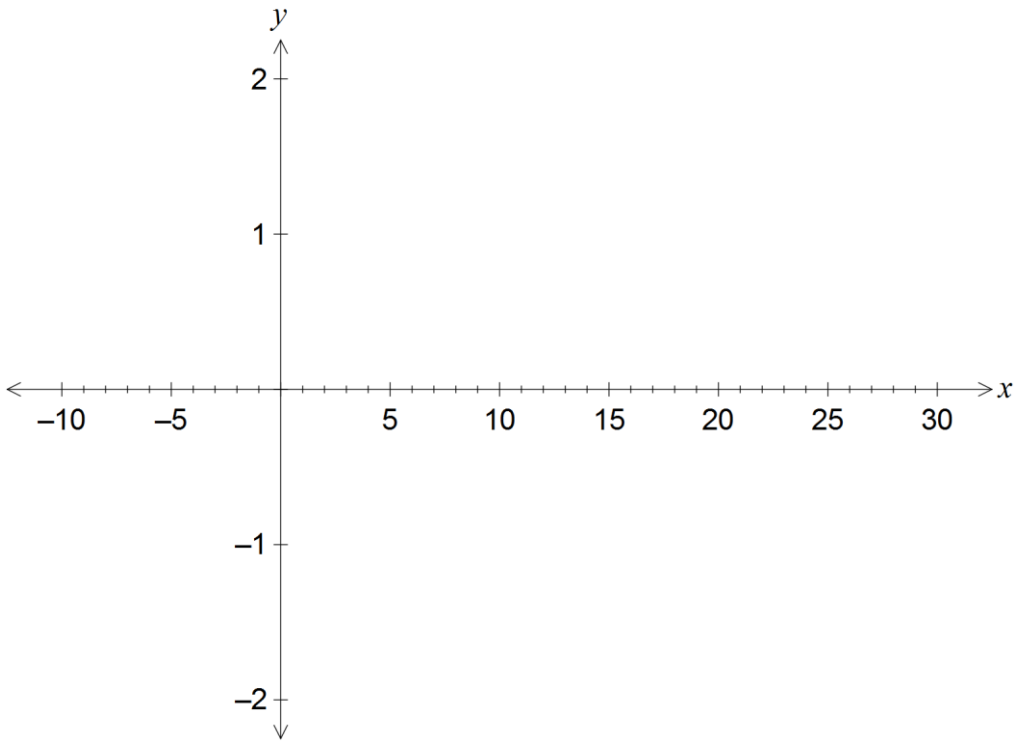
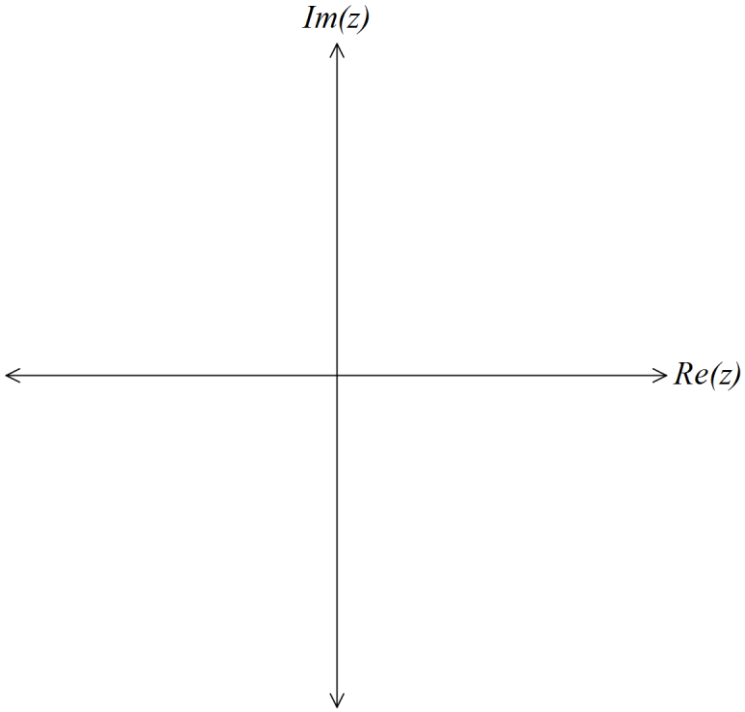
(e) On the axes below, sketch the path taken by the particle for $1 \leq t \leq 10$, clearly indicating the position of the particle at the start and end of this interval. (3 marks)



(f) Determine the length of the path travelled by the particle between $t = 1$ and $t = 10$. (2 marks)

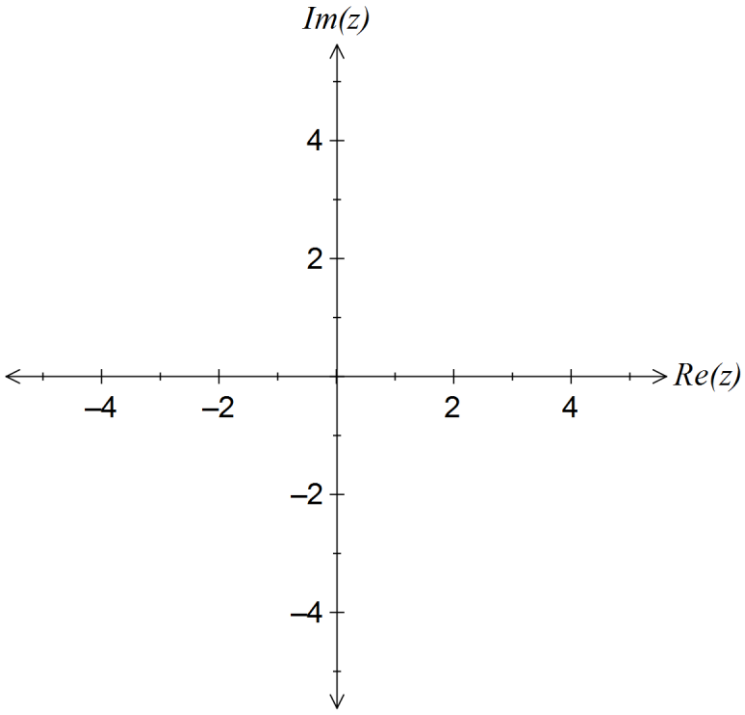
Question 19 **(7 marks)**

(a) Shade the region satisfying the complex inequality $|z-i| > |z-4+3i|$ on the Argand diagram below. (3 marks)



(b) Consider the two complex numbers given by $u = 3-3i$ and $v = -i$. Sketch each of the following sets of points in the complex plane.

(i) $|z-u|+|z-v| = |u-v|$. (2 marks)



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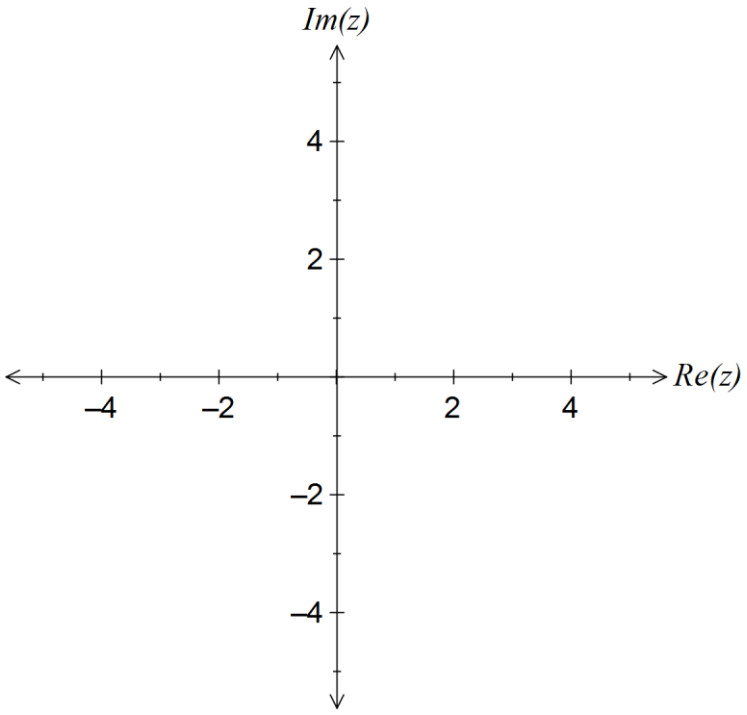
(ii) $|z-v|+|u-v|=|z-u|$. (2 marks)



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